Maths**Practice**

Edexcel GCSE
Mathematics 1MA1

Foundation

Step-by-step guidance and practice

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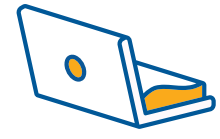

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PREFACE

A crisp, clear and accessible guide to every maths topic, this is everything you need to ace the three exams in this course and beam with pride. Each topic is clearly presented in a format that is clear, approachable and as concise and simple as possible.

Each section of the specification is clearly indicated to help you cross-reference your revision. The checklist on the contents pages will help you keep track of what you have already worked through and what's left before the big day.

We have included worked example questions with answers for each objective. This is followed by a series of related questions that gently increase in their level of challenge. You can check your answers against those given at the end of the book.

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THE SCIENCE OF LEARNING AND REVISION

Variation theory

Procedural variation is the gradual change from one question to the next, in order to subtly increase difficulty. It focuses on not only what changes, but on what does not change. This enables misconceptions and misunderstandings to be identified more easily when only one variable is altered. Seeing these differences, rather than seeing sameness, better enables pupils to reason and make connections.[1]

Retrieval of information

Retrieval practice encourages students to come up with answers to questions.[2] The closer the question is to one you might see in a real examination, the better. Also, the closer the environment in which a student revises is to the 'examination environment', the better. Students who had a test 2–7 days away did 30% better using retrieval practice than students who simply read, or repeatedly reread material. Students who were expected to teach the content to someone else after their revision period did better still.[3] What was found to be most interesting in other studies is that students using retrieval methods and testing for revision were also more resilient to the introduction of stress.[4]

Ebbinghaus' forgetting curve and spaced learning

Ebbinghaus' 140-year-old study examined the rate in which we forget things over time. The findings still hold true. However, the act of forgetting things and relearning them is what cements things into the brain.[5] Spacing out revision is more effective than cramming – we know that, but students should also know that the space between revisiting material should vary depending on how far away the examination is. A cyclical approach is required. An examination 12 months away necessitates revisiting covered material about once a month. A test in 30 days should have topics revisited every 3 days – intervals of roughly a tenth of the time available.[6]

Summary

Students: the more tests and past questions you do, in an environment as close to examination conditions as possible, the better you are likely to perform on the day. If you prefer to listen to music while you revise, tunes without lyrics will be far less detrimental to your memory and retention. Silence is most effective.[5] If you choose to study with friends, choose carefully – effort is contagious.[7]

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CONTENTS AND CHECKLIST

Section 11 Shapes and transformations

Section 12 Sequences

Section 13 Proportion

Section 14 Data

Section 15 Properties of shapes

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Section 17 Further graphs

Section 18 Geometry

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Section 20 Trigonometry

Section 21 Statistics

Section 22 Probability diagrams

Section 23 Mensuration

Section 24 Applications of Ratio

Section 25 Further equations

FOUNDATION TIER

Mathematics (1MA1)

This qualification is assessed over three examination papers.

Specification coverage

The content for assessment in each paper will be drawn from each of the six areas of mathematics:

1. Number
2. Algebra
3. Ratio, proportion and rates of change
4. Geometry and measures
5. Probability
6. Statistics

Assessment

Three written exams: 1 hour 30 minutes

Each 80 marks

All questions are mandatory

Each paper is 33.33% of the qualification grade

Calculators are permitted in Papers 2 and 3 only.

Assessment overview

Each paper will consist of a range of question types.

The total mark across the three equally weighted Foundation tier papers is used to form a combined GCSE grade from 1 to 5.

SECTION 1

INTEGERS

1.1 INTEGERS AND PLACE VALUE

N1 N2

Integers

Integers are whole numbers. They can be positive, zero or negative.

e.g. 6, –10, 0, 107 **are** integers

2.3, $\frac{1}{2}$, –2300.9 are **not** integers

Objectives

Understand and use place value

The value of a digit depends on its position or **place value**.

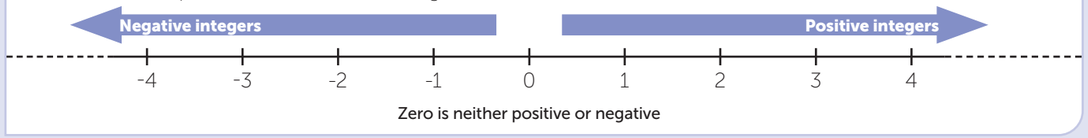
e.g. The number 4603 can be shown as:

thousands 1000	hundreds 100	tens 10	units 1
4	6	0	3

Place integers on a number line

Number line

Numbers can be placed on a number line. Negative numbers are numbers below zero.



Zero is neither positive or negative

Sort integers into ascending or descending order

Ascending order

Numbers sorted in ascending order up go up in size.

e.g. –11, –3, 15, 34, 101 are sorted in ascending order.

Descending order

Numbers sorted in descending order go down in size

e.g. 9, 3, 0, –5 are sorted in descending order.

Practice questions

1. a) In the number 16 289 what is the value of the 6?
   b) Write the number 30 209 in words.
   c) Write the number two million, one thousand and thirty six as a number.

2. This is a list of numbers: 8, 3, –9, 0, –2, 1, –4, –6, 2
   a) Draw a number line with a scale from –10 to 10. Mark the position of each number on the number line.
   b) Write these numbers in descending order.
   c) All the numbers are doubled. What effect will this have on the descending order?

3. Tom has five numbered cards.

   a) Write down the largest number that he can make using all the cards once only.
   b) Write down the largest possible number less than 50 000, that he can make using all the cards once only.
   c) Write down the smallest possible number that is greater than 20, 000 that he can make using all the cards once only.

4. Jodie has tried to write these numbers in ascending order.
   She has made mistakes. Spot the mistakes and correct them.
   a) 4443, 30 400, 44 104, 44 033, 400 300
   b) 280 880, 28 888, 208 088, 8882, 888

5. Here is a map of the British Isles showing the temperatures on 1st December.
   List the temperatures in ascending order.

6. A three-digit number has a tens digit which is two times as big as its hundreds digit. It has a zero units digit.
   Write down the possible numbers it could be.

7. A three-digit number has one repeated digit. Its digits are all prime numbers. It is an even number greater than 700.
   Write down the possible numbers it could be.

8. Use these clues to find the missing number.
   - There is a 2 in the ten thousand and tens place.
   - The units digit is 3 times the digit in the ten thousand place.
   - The digit in the thousands place is 2 less than 5.
   - The digit in the hundred thousand place is 2 more than half of the units digit.
   - The digit in the tens place is 2 more than the digit in the hundreds place.

9. Numbers are written on four cards as shown below. Two of the cards are blank.

   a) Choose numbers to write on the blank cards so that the six cards can be used to complete a number pattern.
   b) Write the six numbers in ascending order.

10. A three-digit number has digits which are ascending consecutive numbers.
    Another three-digit number has digits which are descending consecutive numbers.
    The difference between the two numbers is 24.
    What are the two numbers?

1.2 NEGATIVE INTEGERS

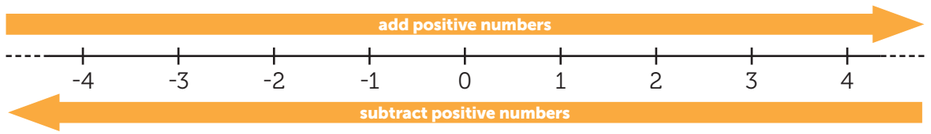
Objectives

Add and subtract positive and negative integers

Adding and subtracting integers

A number line can help when adding and subtracting integers.

Move up the line when adding and down the line when subtracting a positive number.



Adding a negative number is the same as subtracting a positive number. **e.g.** –5 + (–8) = –5 – 8 = –13

Subtracting a negative number is the same as adding a positive number. **e.g.** –5 – (–8) = –5 + 8 = 3

Use a calculator to evaluate arithmetic operations involving positive and negative integers

Calculators

A calculator can be used to check an answer is correct. You do not have to put brackets around negative numbers.

e.g. –124 – –213 = 89

The table shows the temperatures in two different cities on a Monday.

Work out the difference in temperature between Athens and Oslo on that Monday.

Athens	7°C
Oslo	–14°C

Difference in temperature is

7 – (–14) = 7 + 14

= 21°C

Practice questions

1. Work out the answers to these without a calculator.
   a) 13 – 20 b) –10 + 18 c) –10 – 2 d) 8 – 20
   e) –6 – 4 f) –4 + 1 g) –7 + 9 h) –21 + 10

2. Work out the answers to these without a calculator.
   a) 13 + –7 b) –10 + –1 c) –15 + –2 d) 10 + –20
   e) –12 – –3 f) 4 – + 1 g) 9 – – 7 h) –31 – – 8

3. Work out the answers to these using a calculator.
   a) 137 + – 198 b) –509 – – 237 c) –2489 – –379 d) –3478 – + 9691

4. Place two or three of the numbers 7, –11 and –3 into each of of the following, to make the calculation correct.
   a) _____ + _____ = 4 b) _____ – _____ = 18
   c) _____ – _____ = – 8 d) _____ – _____ – _____ = 1

5. Tom says that –10 – 24 = 34. By referring to a number line, give a reason why Tom is wrong.
   Give the correct answer to the calculation.

6. At 6pm the temperature is 2°C. By midnight, the temperature drops to –8°C.
   By how many degrees did the temperature fall?

7. Work out the missing numbers in each calculation.
   Use a calculator to check your answers.
   a) 297 + ____ = 129 b) 1032 + _____ = 1006
   c) ____+ –358 = 1008 d) ____ + – 762 = –2101

8. The table shows the surface temperatures of different planets.

	Minimum surface temperature (°C)	Maximum surface temperature (°C)
Mercury	–170	449
Mars	–125	20
Earth	–89	58

   a) What is the difference between the minimum temperatures on Mercury and on Earth?
   b) What is the temperature difference on Mars?
   c) Which planet has the greatest temperature difference? What is it?

9. In a magic square all the rows, columns and diagonals add up to the same total. Complete the magic square.

1		
–16	–7	
		–15

10. In an addition pyramid each number is the sum of the two numbers beneath it.
    Copy and complete these addition pyramids.

a)

3	–21	–6

b)

	–2	
–8		–6

c)

	–20	
		–11
–16		

1.3 CALCULATING WITH NEGATIVE INTEGERS

Objectives

Multiply and divide positive and negative integers

You can use these rules when multiplying and dividing positive and negative integers.

positive × positive = positive	positive ÷ positive = positive
positive × negative = negative	positive ÷ negative = negative
negative × positive = negative	negative ÷ positive = negative
negative × negative = positive	negative ÷ negative = positive

e.g. $3 \times -6 = -18$ $\quad -5 \times 7 = -35$
$-24 \div 6 = -4$ $\quad -55 \div -5 = 11$

Use the symbols =, ≠, <, >, ≤, ≥

Symbols such as: =, ≠, <, >, ≤, ≥, can be used to make number sentences true.

< less than
> greater than
= equal to
≤ less than or equal to
≥ greater than or equal to
≠ not equal to

e.g. $-6 > -10$ $\quad 5 - 7 < 7 + -7$
$15 - 20 \neq 15 \div -5$ $\quad 4 \times -3 = -4 \times 3$

Use a calculator to evaluate arithmetic operations involving positive and negative integers

Practice questions

Do not use a calculator with these questions unless instructed to do so.

1. Work out these calculations.
   a) 3×-6 b) $-32 \div 4$ c) -9×-6 d) $72 \div -6$
   e) -2×6 f) $24 \div -8$ g) -8×-6 h) $-100 \div -5$

2. Decide if each of these calculations is true or false. If the statement is false, correct it.
   a) $9 \times -6 = 54$ b) $-48 \div -4 = 12$ c) $16 \div -4 = 4$ d) $-7 \times -4 = -28$

3. Use a calculator to work out each of these calculations.
   a) 59×-38 b) -238×-45 c) $2960 \div -80$ d) $-1416 \div -12$

4. The temperature of a metal changes by −3°C every hour.
   What will the total change in temperature be after 6 hours?

5. Match each calculation with its answer. (12, −12, 48, −48)
   a) 6×-8 b) $36 \div -3$ c) $-24 \div 2$ d) -12×-4 e) -12×-1

6. Work out these calculations and write the answers in ascending order.
   a) 12×-11 b) $-36 \div 6$ c) $125 \div -5$ d) 0×-10 e) $-200 \div 2$

7. Here are some number cards.

−6	**4**	**−7**	**1**	**9**

   a) Which two cards when multiplied give the largest possible answer?
   b) Which two cards when multiplied give the smallest possible answer?

1.4 MULTIPLICATION

Objectives

Understand and use the terms sum, difference, product, quotient

Sum, difference, product, quotient

The sum of two numbers is obtained by adding them.	**e.g.** the sum of 11 and 8 is 19
The difference of two numbers is obtained by subtracting them.	**e.g.** the difference of 15 and 8 is 7
The product of two or more numbers is obtained by multiplying the numbers together.	**e.g.** the product of 2 and −3 is −6
The quotient of two numbers is obtained by dividing one number by the other.	**e.g.** the quotient of 12 and −3 is −4

Use non-calculator methods to complete multiplications

Non-calculator methods for multiplication

The grid method is one example of a non-calculator method for multiplication, **e.g.** work out 34 × 26

×	30	4	
20	600	80	
6	180	24	
total	780	104	**884**

So **34 × 26 = 884**

Traditional **long multiplication** method is another example, **e.g.** work out 34 × 27

34
× 27
238
2
680
1
918

So **34 × 27 = 918**

Practice questions

1 Work out the sum of these numbers:
   a) 11 and 23 b) -1 and 1 c) 105 and 24 d) −6 and −2

2. Work out the difference between these numbers:
   a) 12 and 5 b) 3 and 9 c) −5 and 7 d) 6 and −6

3. Here is a list of numbers: −60, −25, −9, −5, 10, 18
   Choose two numbers from the list so that:
   a) their product is −250 b) their quotient is 5 c) their quotient is the largest positive integer

4. Use a non-calculator method to work out the following calculations.
   Show your working.
   a) 34 × 19 b) 25 × 14 c) 18 × 103 d) 124 × 22

3.7 POWERS AND ROOTS

Objectives

Use index notation and the index laws when multiplying or dividing algebraic terms

Laws of indices

Algebraic terms follow the rules of indices.

e.g. $y^2 \times y^3 = y^5$ When multiplying index numbers with the same base, add the indices

$x^7 \div x^3 = x^4$ When dividing index numbers with the same base, subtract the indices

$(p^5)^2 = p^{10}$ When raising a number index form to a power, multiply the indices

Surds

A value which is expressed as a root, such as $\sqrt{7}$ is called a **surd**.

A surd cannot be simplified to remove the root.

e.g. $\sqrt{13}$ is a surd

$\sqrt{25}$ is not a surd because it simplifies to 5

Simplify and manipulate simple algebraic expressions involving powers and roots

In the expression $3x^2 + 2x - x^3$ there are no like terms.

The indices must match up to have like terms.

Like terms can be simplified.

e.g. Simplify $3b^2 + 7b^3 - 5b + 2b^2$
Like terms are $3b^2$ and $2b^2$
$= \mathbf{5b^2 + 7b^3 - 5b}$

e.g. Simplify $4x^2 \times 5x^4 = 4 \times 5 \times x^2 \times x^4$
Use rules of indices to combine the powers
$= \mathbf{20x^6}$

e.g. Simplify $10y^7 \div 5y^3 = \frac{10y^7}{5y^3}$
Use rules of indices to combine the powers
$= \mathbf{2y^4}$

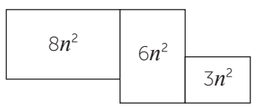
e.g. Simplify $(3p^6)^2 = 3p^6 \times 3p^6$
Use rules of indices to combine the powers
$= \mathbf{9p^{12}}$

Practice questions

1. Write down like terms and then simplify each of the expressions.
   a) $x + 3x^2 + 5x^2 + 4x$ b) $7y^2 - 3y^3 + y^2 - 2y^3 + y$

2. Kim and Laura are looking at the algebraic expression $4d^2 + 5d^4$
   Kim says that the expression cannot be simplified. Laura says it can. Who is correct? Explain your answer.

3. Find the total area of the compound shape below.
   The area of each section is given.

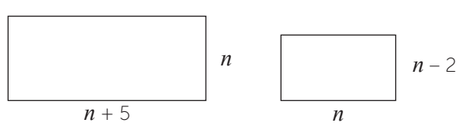


4. Simplify these algebraic expressions.
   a) $5c^3 \times c^4$ b) $2w^4 \times 4w^2$ c) $4p^5 \div 2p$ d) $2d \times 9d^4 \div 3d^2$

5. Identify which of these are surds.
   a) $\sqrt{4}$ b) $\sqrt{17}$ c) $\sqrt{1}$ d) $2\sqrt{3}$

6. Simplify the expression: $p(p - 5) + p^2(4 - p)$

7. Write down an expression for the difference between the areas of the two rectangles.
   Simplify your answer.



8. Expand and simplify the following.
   a) $2x(4 + 3x) - x(2 + x)$ b) $6pq^2(p - 3q)$ c) $2m^2(3m - 1) - 5m(4 - 7m)$

SECTION 4
DECIMALS

N1 N2

4.1 DECIMAL PLACE VALUE

Objectives

Understand and use place value in decimals

Decimal numbers have digits whose value depends on their place value or position

e.g. The number 4.603 can be shown as:

units		tenths	hundredths	thousandths
1		0.1	0.01	0.001
4	●	6	0	3

e.g. The number 0.346 has three tenths, four hundredths and six thousandths.

Order positive and negative decimal numbers

Decimal numbers can be written in ascending or descending order.

Write zero place holders when ordering decimals with different numbers of decimal places.

e.g. When ordering 8.307, 8.3, 8.07, 8.37, consider 8.370, 8.300, 8.070, 8.307.
In ascending order: 8.070, 8.300, 8.307, 8.370

Use a number line

Practice questions

1. Draw out each number line and mark on each number.
   Use >, < or = to make a correct number statement.
   a) 0.13 ___ 0.2

   b) 0.07 ___ 0.065

   c) 0.1000 ____ 0.1

   d) 1.329 ___ 1.4

2. Write down the smallest number in each set.
   a) 2.78, 2.709, 2.8 b) −1.10, −1.4, −1.179 c) −2.35, −2.204, −2.1

3. Write each set of numbers in ascending order.
   a) 4.8, 4.31, 4.2, 4.35, 4.12, 4.09 b) 12.34, 14.85, 12.5, 13.61, 13.7
   c) −0.06, −0.11, −0.27, 0.35, −0.43 d) −1.203, −1.281, 1.2, −1.29, −1.243

SECTION 5

MEASURES

5.1 ESTIMATING ANSWERS

N14 N15

Objectives

Estimate answers to calculations by rounding numbers to 1 significant figure

The answer to a calculation can be estimated by approximating each of the numbers in the calculation.

Often the numbers are approximated to 1 significant figure.

e.g. Estimate the answer to $\frac{0.95 \times 3.207}{1.12 + 2.8}$

by rounding each number to 1 significant figure.

$0.95 \approx 1$ $3.207 \approx 3$ $1.12 \approx 1$ $2.8 \approx 3$

So $\frac{0.95 \times 3.207}{1.12 + 2.8} \approx \frac{1 \times 3}{1 + 3}$ **Estimate** $= \frac{3}{4}$

Round to a sensible degree of accuracy

When giving a final answer to a calculation, give a degree of accuracy which is sensible.

e.g. A room of total area 44 m^2 is to be painted. A tin of paint covers 15 m^2

The number of tins required would be $42 \div 15 = 2.933...$ tins.

It is sensible to round this to **3 tins**, since only whole tins of paint can be bought.

Estimate lengths by comparing with known lengths

We can use known measurements to approximate unknown ones.

e.g. A handspan is approximately 20 cm

This can be used to estimate the length of the diagonal of a TV screen.

Use inequality notation to indicate error intervals due to truncation or rounding

A **truncated number** is approximated by removing all decimal digits to the right of the required accuracy level.

e.g. Truncate the number 3.509 73 to 3 d.p.

Look at the digit in the third decimal place.
3.509 73

Remove all digits to the right of the third decimal place
3.509

Notice how the 7 does not make the 9 round up.

Error intervals can be written for truncated numbers too.

e.g. The width of an envelope is given as 14.2 cm truncated to 1 d.p.

The error interval of the width of the envelope is
14.2 cm ≤ actual width < 14.3 cm

Practice questions

1. Write each of the following numbers rounded to 1 significant figure.
   a) 18.5 b) 7.3 c) 135 d) 0.029 e) 0.006 62

2. Write each of the following numbers truncated to 1 decimal place.
   a) 14.59 b) 3.084 c) 0.271 d) 13.829 e) 0.006 62

3. By approximating each number, estimate the answer to each of these calculations.
   a) $\frac{6.6 \times 9.4}{7.4}$ b) $\frac{11.6 \times 3.87}{5.7}$ c) $\frac{14.2 \times 9.6}{19.7}$ d) $\frac{18.5 \times 2.8}{6.42}$

4. Use approximation to estimate the answer to each of these calculations.
   a) $\frac{(43.2 - 8.9)}{2.8}$ b) $(6.7 + 4.8) \times (10.45 - 2.7)$ c) $\frac{(58 - 19)}{(1.2 \times 3.9)}$

5. Mark's handspan is approximately 11 cm wide.
   The length of his guitar is approximately 8 handspans.
   Estimate the length of his guitar in centimetres.

6. Rachel has an arm span of 1.8 m.
   She estimates the length of a garden hedge as 7 of her arm spans.
   Estimate the length of the garden in metres.

7. The reaction time for a science experiment is 35.7 seconds, rounded to 1 d.p.
   Give the error interval of the time of the experiment.

8. Ann measures the length of her dog walk as 4.8 km truncated to 1 decimal place.
   Give the error interval of the length of her dog walk.

9. Work out the each of these calculations on a calculator.
   (i) Write down all the digits in the answer on your calculator display.
   (ii) Write your answer to a suitable degree of accuracy.
   (a) $\frac{5.3^3}{2.87 \times 1.76}$ b) $\frac{1.94 \times 5.89}{2.6^2}$ c) $\frac{6.22 + \sqrt{141}}{29}$ d) $\frac{19.7 \times 1.6^3}{\sqrt{39}}$

10. Here is a formula $v^2 = u^2 + 2as$, where:
    v is the final velocity,
    u is the initial velocity,
    a is acceleration and
    s is the distance.
    A car starts from rest and accelerates at 4 m/s^2 over a distance of 100 m.
    Work out the final velocity of the car.
    Give you answer to a sensible degree of accuracy.

R2 G15 N13

5.2 SCALE DIAGRAMS

Objectives

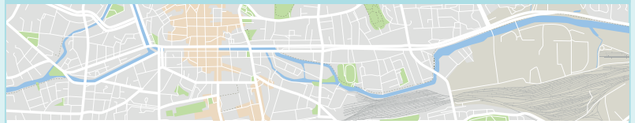
Make an accurate scale drawing from a diagram

A scale diagram represents a place or object.

The distances and lengths of the real places and objects are usually 'scaled down'.

Angles stay the same in a scale diagram.

Know that scale diagrams, including bearings and maps, are 'similar' to the real-life examples



A map is a scale diagram of an area showing various physical features

Use and interpret scale drawings

The scale is used to work out actual distances or distances on the scale diagrams.

e.g. A scale of 1 cm to 50 cm is used to build a model of a helicopter.
The rotor blades on the model are 8 cm long.
Using the scale, the length of the real blades is 8 × 50 = **400 cm or 4 m long**

The real length of the helicopter is 30 m.

Using the scale, the length of the model helicopter is 3000 ÷ 50 = **60 cm**

Explain: Why is the real length converted into centimetres first?

Find the scale

The scale can be found by comparing a distance on the scale drawing with the same distance in real life.

e.g. The distance between two towns is 50 km in real life. On a map it is 2.5 cm.

Work out the scale of the map.

2.5 cm is equivalent to 50 km

1 cm is equivalent to 50 ÷ 2.5 = 20 km. **Scale is 1 cm to 20 km**

Practice questions

1. Triangle ABC is right-angled at B. AB is 6 m and BC is 10 m.
   a) Draw a scale diagram of the triangle using a scale of 1 cm to 1 m.
   b) Measure the length of AC on the scale drawing
   c) Write down the actual length of AC on the triangle.

2. The scale of a drawing is 12 mm to 3.5 m. Copy and complete the table.

Scale length (mm)	Real-life length (m)
24	
	17.5
	28
84	

3. A model of a yacht is built to a scale of 1 cm to 32 cm.
   a) The model is 60 cm long. What is the actual length of the yacht in metres?
   b) The yacht has a mast which is 6.8 m high. How high is the mast on the model?

4. A scale drawing of an office building shows the building as 24 cm tall. The scale of the drawing is 1 cm to 5 m. Work out how tall the real office building is.

5. A map is drawn with a scale of 1 cm to 2.5 km. Jasmine says this is the same as using a scale of 5 cm to 15 km. Is she correct? Give a reason for your answer.

6. On a scale drawing, a roof beam is shown as 13 mm long. The real beam measures 2.6 m long. What is 1 cm on the scale drawing equivalent to, in real-life?

7. The real distance between two towns is 16 km. On a map, the distance between the towns is 40 cm. What is the scale of the map?

8. A garden is 45 m long. A scale drawing of the garden shows the length as 18 cm. What is the scale of the drawing?

9. An architect builds a model of a proposed new athletics stadium. Her model is built on a scale of 1 cm to 5 m.
   a) On the model, how long will the 100 m running straight be?
   b) On the model, what is the distance around the inside of the 400 m track?
   c) The model long jump run-up measures 5 cm. How long is the real run-up?

10. A rectangular wall measures 18 m wide by 8 m high. The wall has a door in the centre measuring 2 m by 1 m.
    a) Make a scale drawing of the wall and the door. Use a scale of 1 cm to 2 m.
    b) Use your scale drawing to work out the real length of the diagonal of the door. Give your answer to the nearest centimetre.

5.3 BEARINGS

G15 R2

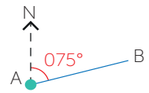
Objectives

Give bearings between points on a scale diagram or map

A **bearing** describes a direction.

All bearings are measured clockwise from North and are given with 3 digits.

e.g. The bearing of B from A is 075°.



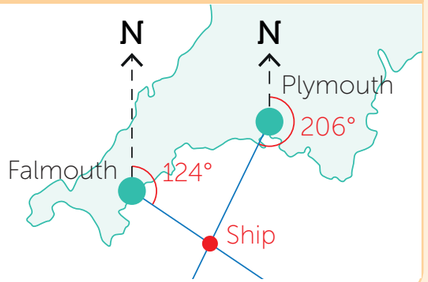
Using bearings

Bearings can be used in scale diagrams.

e.g. The bearing of a ship from Falmouth is 124°.

The bearing of the ship from Plymouth is 206°.

Here is a diagram to show the position of the ship.



Understand clockwise and anticlockwise

An angle can be measured with a turn in a clockwise or anticlockwise direction.

e.g. ↑ rotated through 90° clockwise looks like this →

Make an accurate scale drawing from a diagram, including for solving bearings problems

SECTION 7

STRAIGHT LINE GRAPHS

7.1 WORKING WITH COORDINATES

A8

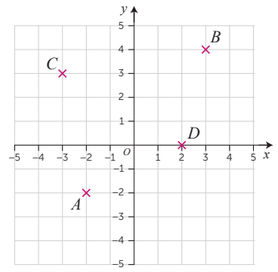
Objectives

Plot and read coordinates in all four quadrants

All coordinates are written in the form (x, y) where:

x is the horizontal movement from (0, 0) and
y is the vertical movement.

e.g. Point A is at (–2, –2), B is at (3, 4), C is at (–3, 3) and D is at (2, 0)



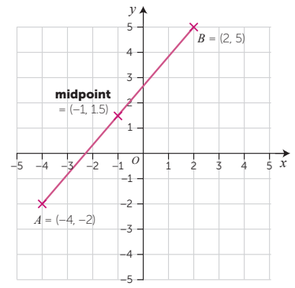
Plot and read coordinates in all four quadrants

Midpoint of a line segment

The midpoint has coordinates exactly in the middle of the coordinates of the two end points.

We calculate the midpoint between (x_1, y_1) and (x_2, y_2) by using $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

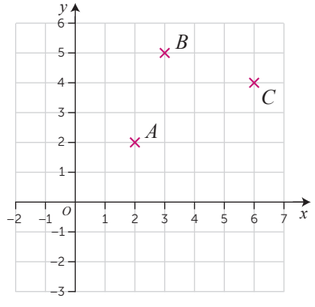
e.g. Midpoint of AB is $\left(\frac{-4 + 2}{2}, \frac{-2 + 5}{2}\right) = (-1, 1.5)$



Solve problems involving coordinates

Practice questions

1. Look at the coordinate grid.
   a) Write down the coordinates of points A, B and C.
   b) Write down the coordinates of a fourth point D, which forms a square ABCD.

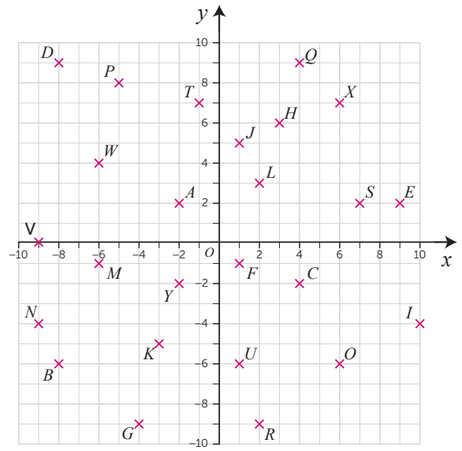


2. Using this letter grid, write down the letters at these points.
   a) (2, 3) b) (–6, –1)
   c) (–9, 0) d) (1, –6)
   e) (–5, 8) f) (–4, –9)

3. Write down the coordinates for these letters.
   a) W b) X
   c) D d) E
   e) B f) C

4. Write down the name of each line segment on the letter grid, which has these midpoints.
   a) (1, –2) b) (–6, 1.5)
   c) (4, 5) d) (–5, 5.5)

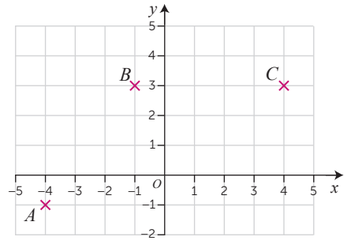
5. Find the coordinates of the midpoint of each of these line segments on the letter grid.
   a) OX b) SE c) AH d) BR



6. (9, 4) and (1, 2) are connected with a line segment.
   a) Find the coordinates of the midpoint of the line segment.
   b) Write down the coordinates of two more points, with positive integer coordinates which have the same midpoint.

7. Find the midpoint of the following pairs of coordinates.
   a) (0, 0) and (6, 8) b) (–4, 10) and (0, 0) c) (2, 6) and (–2, 4) d) (–1, 3) and (–7, 4)

8. Look at this grid.
   a) Write down the coordinates of the point D that forms a rhombus ABCD.
   b) Work out the midpoint of AC and of BD
   c) Comment on what you notice.



9. Gary says that the midpoint of (3, 4) and (p, q) is (–1, 1.5)
   Find the values of p and q.

10.6 MAP SCALES

Objectives

Use and interpret maps

The scale of a map or scale drawing can be given using ratio.

The ratio shows how many units in real life one map unit represents.

e.g. 1 : 200 means that 1 unit on the map represents 200 units in real life.

e.g. 1 : 100 000 means that 1 unit on the map represents 100 000 units in real life.

This can be converted to actual units. Take centimetres as the units.

1 cm : 100 000 cm 1 cm : 1000 m 1 cm : 1 km

Convert scales as ratios to units

The ratio showing the scale on a map can be used to find real or map distances.

e.g. The scale of a map is 1 : 200 000. The real distance between two cities is 156 km. Find the map distance.

The scale is 1 : 200 000 or 1 cm : 2 km

The map distance is 156 ÷ 2 = **78 cm**

e.g. The scale of a map is 1 : 10 000

The map distance between two villages is 16 cm. Find the real distance.

The scale is 1 : 10 000 or 1 cm : 100 m

The real distance is 16 × 100 = **1600 m or 1.6 km**

Estimate distances on maps

Practice questions

1. On a map, 1 cm represents 8 km.
   What distance on the map will represent a real distance of:
   a) 16 km b) 32 km c) 4 km d) 12 km

2. On a map, 1 cm represents 5 km.
   Work out the real distance between two places if their map distance is:
   a) 2 cm b) 4.8 cm c) 6.5 cm d) 12.2 cm

3. The scale of a map is 1 : 500 000
   On the map the distance between two towns is 6.2 cm.
   Work out the real distance between the two towns. Give your answer in km.

4. The scale of a map is 1 : 100 000
   Work out the map distance between two towns which are 19 km apart.

5. Match the map scales with the correct map ratio.

   a) 1 : 35 000
   b) 1 : 350 000
   c) 1 : 100 000
   d) 1 : 10 000

   i) 1 cm to 3.5 km
   ii) 1 cm to 350 m
   iii) 1 cm to 10 km
   iv) 1 cm to 100 m
   v) 1 cm to 1 km

6. Write each scale as a map ratio.
   a) 1 cm to 2 km b) 3 cm to 1.5 km c) 1 cm to 300 m d) 5 cm to 1 km

7. The distance between two towns is 6 km. On a map the distance between the towns is 30 cm.
   Work out the ratio scale of the map.

8. This is the map of the UK.
   It has a scale of 1 cm to 50 km.
   Estimate the real distance between:
   a) Exeter and Aberdeen
   b) Belfast and London
   c) Cardiff and Norwich

9. Georgia wants to draw a map of her school which covers an area of 1 km by 1.3 km.
   Her map must fit onto a piece of paper measuring 28 cm square.
   Suggest a suitable scale for her map. Give your answer as a ratio.

10. The dimensions of a park on the city map are 12 cm by 15 cm. The area of the park is 1.8 km^2.
    Which **one** of these map ratios has been used: 1 : 10 1 : 100 1 :1000 1 : 10 000
    Give reason for your answer.

SECTION 11

SHAPES AND TRANSFORMATIONS

11.1 ANGLE PROPERTIES OF LINES

G1 G3

Objectives

Use two-letter notation for a line and three-letter notation for an angle

Figure 1 shows two line segments AB and DM. M lies on segment AB.
Angle x can be written as ∠AMD or ∠ DMA.
Angle y can be written as ∠DMB or ∠BMD.
Angles that meet at a point on a straight line add up to 180°. ($x + y = 180°$)

Figure 2 shows two line segments PQ and RS, which intersect at point X.
Angles that meet around a point add up to 360° ($a + b + c + d = 360°$)
Angles that are opposite each other when two straight lines intersect are called **vertically opposite angles**.
Vertically opposite angles are equal: $a = b$ and $c = d$

When a straight line crosses a pair of parallel lines, it makes a set of **corresponding angles**, and a set of **alternate angles**.
Corresponding angles lie in corresponding positions on each parallel line.
Corresponding angles are equal to each other:
In **Figure 3**: $w = u, x = v, z = t$ and $y = s$
Alternate angles lie inside the parallel lines, on opposite sides of the line that crosses them.

Alternate angles are equal to each other.
In **Figure 3**: $y = u$ and $x = t$

Figure 1

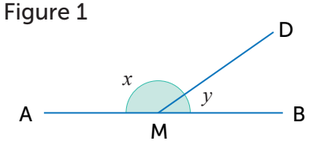


Figure 2

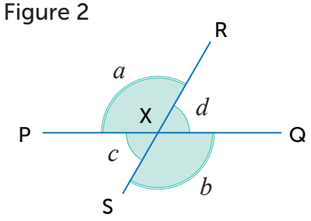


Figure 3

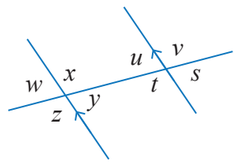
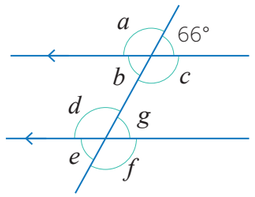


Figure 4



e.g. In **Figure 4**, find the angles marked with a letter in this diagram:

a = 180 – 66
= 114° (angles on a straight line add up to 180°)
b = 66° (vertically opposite to 66°)
c = 114° (vertically opposite to a)
d = 114° (alternate to c)
e = 66° (corresponding to b)
f = 114° (corresponding to c)
g = 66° (alternate to b)

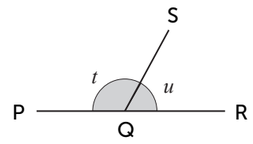
Recall and use properties of angles at a point, angles at a point on a straight line, right angles and vertically opposite angles

Find missing angles in parallel lines using properties of corresponding and alternate angles

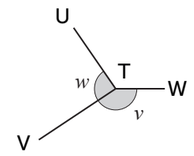
Practice questions

1. Use the three-letter notation to describe angles t, u, v, w, x and y in the following diagrams:

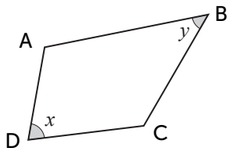
a)



b)

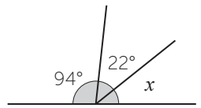


c)

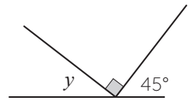


2. Work out the value of each angle marked with a letter.
Give reasons for your answers.

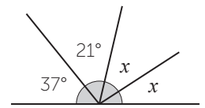
a)



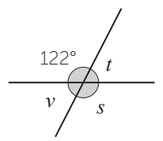
b)



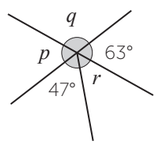
c)



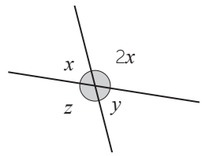
d)



e)

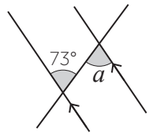


f)

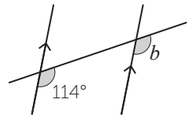


3. For each diagram, find the lettered angles .Give reasons for your answers.

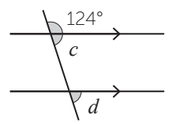
a)



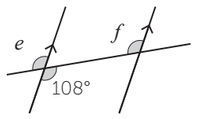
b)



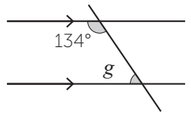
c)



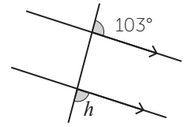
d)



e)

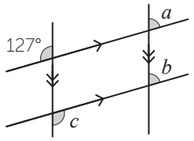


f)

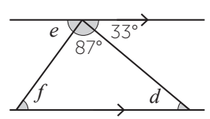


4. For each of the following diagrams, work out the size of the lettered angles.
Give reasons for your answers.

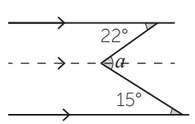
a)



b)



c)



11.5 ENLARGEMENTS AND SIMILARITY

Objectives

Understand that similar shapes are enlargements of each other and angles are preserved

Similarity

When two shapes have the same shape, including the same angles, but are different sizes by length then they are **similar**. A shape and its enlargement are always similar.

Enlarge objects using simple integer scale factors

Here is Radcliffe Camera in Oxford. Every length on the second picture is three times that of the first picture, so the second picture is an enlargement of the first by scale factor 3.

In an enlargement, lengths and areas change, but angles do not.

1 unit

3 units

Identify the scale factor of an enlargement of a shape as the ratio of the lengths of two corresponding sides:

To find the scale factor of an image, find the ratio of one length to the corresponding length on the object.

$$\text{Scale Factor} = \frac{\text{Length of side on image}}{\text{Length of corresponding side on object}}$$

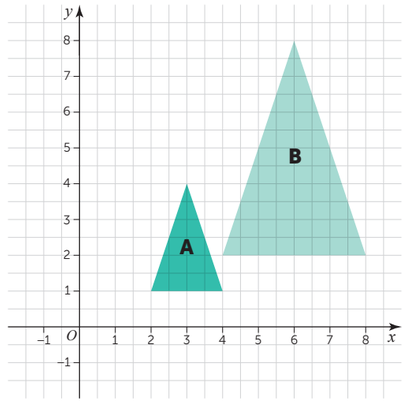
e.g. Shape B is an enlargement of shape A.

To find the scale factor enlargement from A to B.

The base of A is 2 units, the base of B is 4 units.

The scale factor from A to B $= \frac{4}{2}$

$= 2$

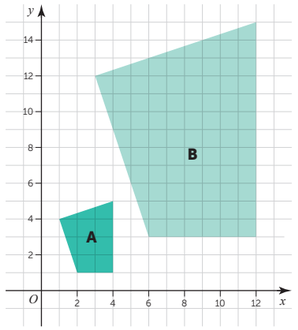


Practice questions

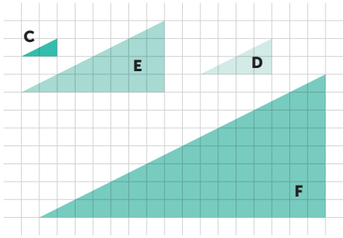
1. A rectangle has length 24 cm and width 15 cm. If it is enlarged by scale factor 4, what will its new length and width be?

2. A photo which is 12 cm x 18 cm is enlarged by scale factor 3.
   What are the dimensions of the enlarged photo?

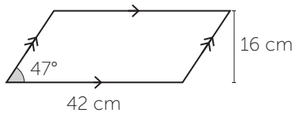
3. Work out the scale factor of the enlargement that transforms shape A onto shape B.



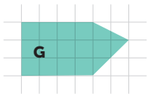
4. Here are four triangles, C, D, E and F.
   Work out the scale factor of:
   (a) D from C.
   (b) E from C.
   (c) F from C.
   (d) F from D.



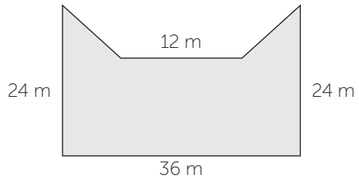
5. A parallelogram has a base of 42 cm, a perpendicular height of 16 cm and its smaller angles are each 47°.
   If the parallelogram is enlarged by a scale factor 5, what will its new measurements be?



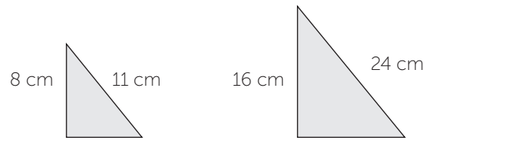
6. Shape G is as shown.
   Draw an enlargement, scale factor 4 and label it H.
   Enlarge H by a scale factor of 2 and label it J.
   What scale factor would enlarge shape G to get shape J?



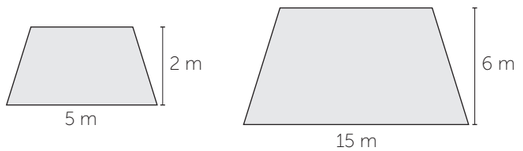
7. An enlarged shape has dimensions as shown.
   It was enlarged from a smaller shape by a scale factor of 6.
   What were the dimensions of the original shape?



8. For each pair of shapes, say if they are similar and give your reasons.
   a)



   b)



SECTION 14

DATA

14.1 CATEGORICAL AND DISCRETE DATA

S2

Objectives

Understand different types of data

Categorical data is non-numerical and is often descriptive. For example, colours of cars.
Discrete data is numerical and takes distinct values. For example, number of students in a team.

Design and use data collection sheets

A common way of collecting data is using a tally chart.

e.g. Complete the tally chart for the following data about the number of hens' eggs stolen by a fox, over a three-week period.

Fill in the frequencies.

1	3	2	3	1	1	0
2	2	0	1	1	2	2
3	1	2	0	2	3	1

Eggs laid	Tally	Frequency
0	III	3
1	𝍸 II	7
2	𝍸 II	7
3	IIII	4

Note that the tallies are grouped so that 𝍸 means five.

Construct and interpret pictograms and bar charts.

e.g. The following data was collected about the lunch preferences of 32 students.

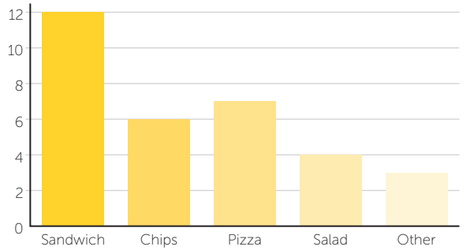
Lunch	Frequency
Sandwich	12
Chips	6
Pizza	7
Salad	4
Other	3

Draw

a) A bar chart b) A pictogram

to represent this data.

Bar chart



Pictogram

Key: represents 4 students

Practice questions

1. The data on the right was collected about people's favourite colours:
B = Blue, O = Orange, R = Red, P = Pink and G = Green.
Draw and complete a tally chart for this data.

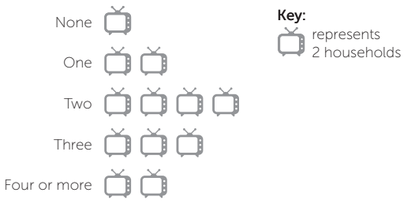
B	O	R	B	G
R	B	O	R	O
O	B	G	B	R
R	G	P	P	B

2. The data below is collected on the football team 24 people support:

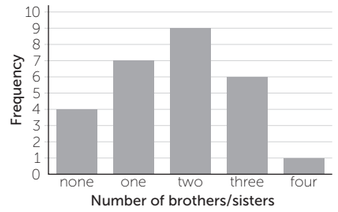
MU	C	A	MC	A	MC	MC	MU	C	MU	C	MC
L	MC	MU	C	MU	MU	MU	MU	L	O	C	O

MU = Manchester United, MC = Manchester City, C = Chelsea, A = Arsenal, L = Liverpool, O = Other
Draw and complete a tally chart for this data. Which team was the most popular?

3. The pictogram represents the number of televisions in households:
   a) How many households had no televisions?
   b) What is the most common number of televisions in this survey?
   c) How many households were surveyed?



4. The bar chart shows the numbers of brothers and sisters the members of a form group have:
   a) How many members of the form have 1 brother or sister?
   b) What is the largest number of brothers or sisters that any member of the form had?
   c) How many people are in the form group?



5. The tally chart represents the money to the nearest five pounds, that a group of people had in their pockets:
Draw a pictogram of this information using the key:

£ represents 3 people

Amount (£)	Frequency
0	3
5	6
10	15
15	12
20	9
> 25	6

6. Jennifer is designing a sheet to collect data on pocket money received by people in her maths class; so far, she has done this:

Name	Pocket money

Explain how Jennifer could make her sheet better.

15.4 CIRCUMFERENCE

Objectives

Recall and use formulae for the circumference of a circle
Give an answer in terms of π

A circle is a shape defined by a set of points which are all the same distance from a given point (the centre).

The circumference of a circle of radius r = $2\pi r$, where π = 3.142 (to 3 decimal places)

The π button on your calculator will use a value to several decimal places. π can be left in the answer, if an exact value is required

e.g. A circle has diameter 19 cm. What is its circumference?
Give your answer in terms of π, and then to 1 d.p.
If the diameter is 19 cm, then the radius = 9.5 cm

Circumference $= 2\pi r$
$= 2 \times \pi \times 9.5$
$= 19\pi$ cm $\Rightarrow$ 59.7 cm (1 d.p.)

e.g. Find the radius of a circle if its circumference is 36.3 m.

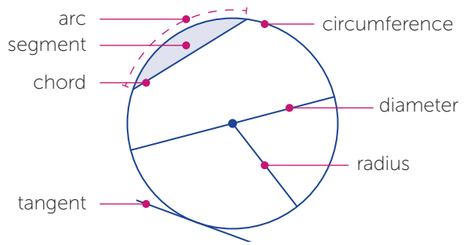
Circumference $= 2\pi r \Rightarrow r = \frac{\text{Circumference}}{2\pi}$

$r = \frac{36.3}{2\pi}$

$= 5.8$ m

Find radius or diameter, given perimeter of a circle

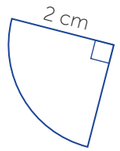
Recall the definition of a circle and identify, name and draw parts of a circle including tangent, chord, arc and segment



Find the perimeters of semicircles, quarter-circles

The perimeter is the distance all the way around a shape. If part-circumferences are required, don't forget to include the other lengths that make up the perimeter.

e.g. A silversmith is basing the design for a pendant on a quarter circle radius 2 cm.
Find the perimeter of the silver pendant.
Give your answer in terms of π.



If it was a full circle, circumference
$= 2 \times \pi \times 2$ $= 4\pi$ cm
Perimeter of pendant $= \frac{4\pi}{4} + 2 + 2$
$= (\pi + 4)$ cm

Practice questions

Use the π button on your calculator in these practice questions.

1. Calculate the circumference of circles with these radii:
   a) 7 cm b) 25 cm c) 16.3 m d) 235.2 mm

2. Calculate the circumference of circles with these diameters:
   a) 12 cm b) 45.3 m c) 82.6 mm d) 143.7 cm

3. If the Earth has a radius of approximately 6400 km, what is the length of the equator?

4. A tennis ball has a diameter of approximately 6.7 cm. What is its circumference?

5. What is the circumference of each of these coins?
   a) 1p coin, with diameter 2 cm b) 2p coin, with diameter 2.6 cm
   c) 5p coin, with diameter 1.7 cm d) 10p coin, with diameter 2.4 cm

6. A rope is tied 8 times around a capstan (cylindrical post). The post is 42 cm diameter.
   How long is the rope around the capstan?

7. The shooting 'circle' on a netball court is a semi-circle of radius 4.9 m
   What is the perimeter of the semi-circle?

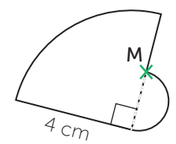
8. A bicycle wheel has a diameter of 630 mm including the inflated tyre.
   a) Calculate the circumference of the wheel.
   b) If the wheel turns 2400 times, what distance has it covered, to the nearest metre?

9. The radius of the stone circle at Stonehenge is approximately 15 m.
   What is the approximate perimeter of the stone circle?

10. Heather cuts a 30 cm diameter pizza into 4 quarters.
    What is the perimeter of each piece?

11. A human wrist can be taken to be roughly circular. Wrist measurements are the circumference of the arm at the wrist.
    a) What is the wrist measurement of Zach whose arm is 70 mm wide at the wrist?
    b) What is the wrist measurement of Jasmine whose arm is 55 mm wide at the wrist?
    c) If Josef has a wrist measurement of 164 cm, how wide is his arm at the wrist?

12. The shape shown is made up of a quarter circle and a semicircle.
    M is the mid-point of the radius of the larger circle.
    Find the perimeter of the shape.
    Leave π in your answer.



SECTION 18

GEOMETRY

18.1 CONSTRUCTIONS

G1 G2

Objectives

Make accurate drawings of triangles and other 2D shapes using a ruler and a protractor

Constructions

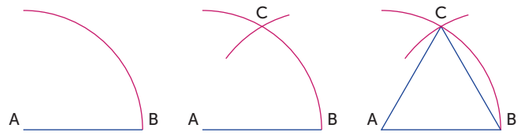
Many diagrams featuring straight lines, angles and 2D shapes can be drawn using only straight edges and compasses. They involve no direct measurement and rely on geometric properties.

e.g. Construct an equilateral triangle on base AB.

a) Draw arc, centre A, radius equal to AB.

b) Draw arc, centre B, same radius to cross at C.

c) Complete the triangle ABC.



Use straight edge and a pair of compasses to do standard constructions: perpendicular bisector of a given line; perpendicular from a point to a line; bisector of a given angle; angles of 90°, 45°

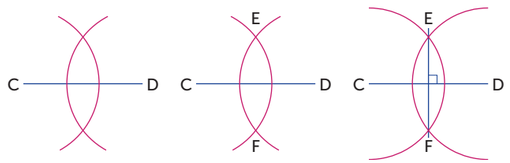
Constructions

e.g. Construct the perpendicular bisector on the line CD.

a) Draw arc, centre C, radius more than half of CD.

b) Draw arc, centre D, same radius to cross at E and F.

c) Join EF which cuts CD at its midpoint.

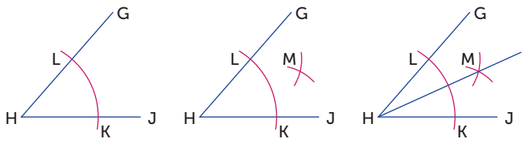


e.g. Construct the bisector of angle GHJ.

a) Draw arc, centre H to cross HJ at K and HG at L.

b) Draw arc, centre K, and draw arc, centre L, same radius to cross at M.

c) Join HM, the angle bisector of GHJ.



Practice questions

1. Draw a straight line and label it LM.
   Using only compasses, construct the perpendicular bisector of LM.

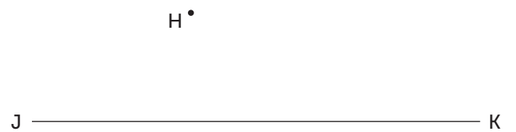
2. Draw two straight lines that meet at a point and label the resulting angle PQR.
   Using only compasses, construct the bisector for angle PQR.

3. Construct the perpendicular at X on the line VW.

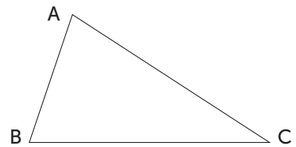
4. Draw a triangle and label the vertices X, Y and Z.
   Construct the angle bisectors for each angle in triangle XYZ.

5. Draw a circle, radius 4 cm and construct a regular hexagon inside it.

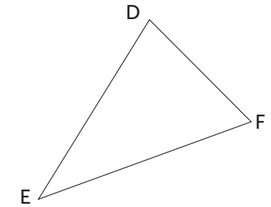
6. Construct a perpendicular from point H to line JK.



7. Copy the triangle ABC.
   Construct the perpendicular bisector for each side of triangle ABC.

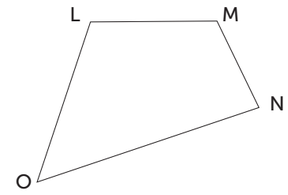


8. Construct the perpendicular from each vertex of triangle DEF to its opposite side.



9. Draw a straight line and label it JK. Construct an angle of 60° on the line JK.
   Then bisect this angle to make two angles of 30° each.

10. Construct the perpendicular bisectors on each side of quadrilateral LMNO.



G6 G19 R12

18.4 SIMILARITY

Objectives

Solve problems to find missing lengths in similar shapes

Two shapes are mathematically similar if corresponding angles are all equal and corresponding sides are in the same ratio.

One shape may be a scaled up, scaled down, rotated or reflected version of the other.

If two shapes are similar, then the scale factor of the enlargement of corresponding sides on the object and image is the same.

e.g. Triangles XYZ and ABC are similar.

XY = 5 cm, YZ = 8 cm, BC = 15 cm and AC = 27 cm.

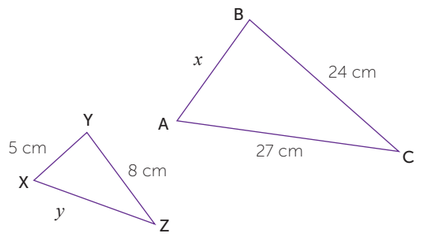
Calculate the length of sides AB and XZ.

Scale factor of the enlargement: $\frac{BC}{YZ}$

$= \frac{24}{8}$ $\Rightarrow 3$;

All sides of XYZ are multiplied by 3: AB $= 5 \times 3$ $\Rightarrow \boldsymbol{x} = 15$ cm

$\boldsymbol{y} \times 3 = 27$ $\Rightarrow \boldsymbol{y} = 9$ cm

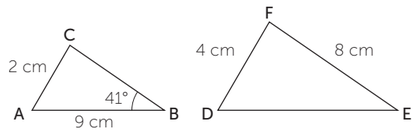


Understand similarity of triangles and of other plane shapes, including all circles or all regular polygons with equal number of sides

Solve angle and length problems using similarity

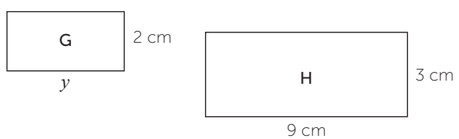
Practice questions

1. Triangles ABC and DEF are similar.
   a) What is the size of angle DEF? Explain your answer.
   b) Work out the length of BC.
   c) Work out the length of DE.

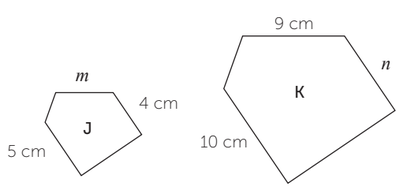


2. One rectangle measures 7 m × 3 m, and another measures 14 m × 7 m.
   Are the two rectangles similar? Explain your reasoning.

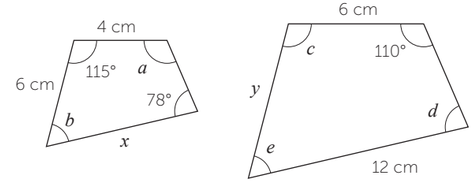
3. Rectangles G and H are similar.
   Work out the value of length y.



4. Pentagons J and K are similar.
   a) What is length $\boldsymbol{m}$?
   b) What is length $\boldsymbol{n}$?

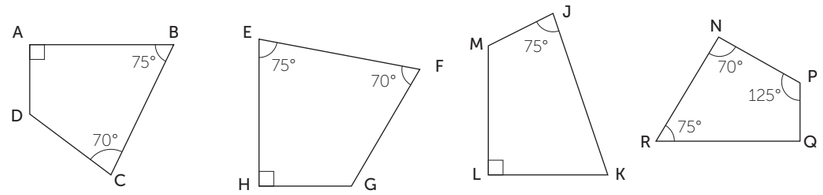


5. These two quadrilaterals are similar.
   Find the angles $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, $\boldsymbol{d}$ and $\boldsymbol{e}$.
   Find lengths $\boldsymbol{x}$ and $\boldsymbol{y}$.



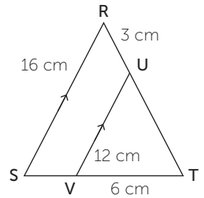
6. In these two similar triangles, the scale factor of enlargement is 3.5
   Find the values of $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$.
   What type of triangles are these?

7. Four quadrilaterals (not drawn to scale) are shown below.

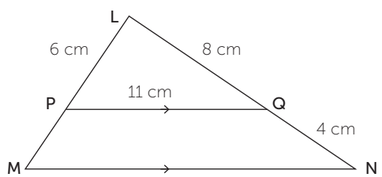


   a) Calculate ∠ADC
   b) Which of the quadrilaterals is not similar to the other three?

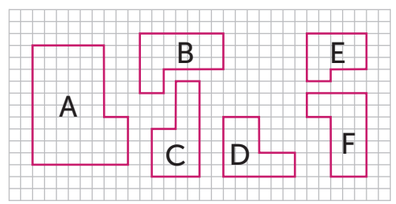
8. In triangle RST, UV is parallel to RS.
   Draw, and label triangles RST and UVT as separate diagrams.
   a) Calculate the length of SV.
   b) Calculate the length of TU.



9. In triangle LMN, PQ is parallel to MN.
   Draw and label triangles LPQ and LMN as separate diagrams.
   a) What is the length of PM?
   b) What is the length of NM?



10. Six shapes are shown below.
    a) Two of the shapes are congruent.
    Write down the letters of these shapes
    b) One of the shapes is similar to A.
    Write down the letter of this shape



ANSWERS

Section 1

1.1 Integers and place value

1. a) 6 000 b) thirty thousand, two hundred and nine c) 2 001 036
2. a) 

   b) 8, 3, 2, 1, 0, –2, –4, –6, –9
   c) It will have no effect on the order since they are all twice as big
3. a) 97 431 b) 49 731 c) 31 479
4. a) 4443, 30 400, 44 033, 44 104, 400 300
   b) 888, 8882, 28 888, 208 088, 280 880
5. –4, –2, 0, 1, 2
6. 480, 360, 240, 120
7. 772 or 722
8. 523 026
9. a) 269 350 and 225 350
   b) 225 350, 236 350, 247 350, 258 350, 269 350, 280 350
10. 567 and 543

1.2 Negative integers

1. a) –7 b) 8 c) –12 d) –12 e) –10 f) –3
   g) 2 h) –11
2. a) 6 b) –11 c) –17 d) –10 e) –9 f) 3
   g) 16 h) –23
3. a) –61 b) –272 c) –2110 d) –13 169
4. a) –3 + 7 b) 7 – –11 c) –11 – –3 d) –3 – 7 – –11
5. Subtracting 24 means to move down the number line from –10, so the answer will be smaller than – 10. The answer is – 34.
6. The temperature fell by 10°C
7. a) –168 b) –26 c) 1366 d) –1339
8. a) 81°C b) 145°C c) Mercury at 619°C
9.

1	–14	–8
–16	–7	2
–6	0	–15

10. a)

	–45	
–18		–27
3	–21	–6

b)

	–2	
–2		0
–8	6	–6

c)

		–20	
	–9		–11
–16		7	–18

1.3 Calculating with negative integers

1. a) –18 b) –8 c) 54 d) –12 e) –12 f) –3
   g) 48 h) 20
2. a) False –54 b) True c) False –4 d) False 28
3. a) –2242 b) 10 710 c) –37 d) 118
4. –18°C
5. a) –48 b) –12 c) –12 d) 48 e) 12
6. a) –132 b) –6 c) –25 d) 0 e) –100
   –132, –100, –25, –6, 0
7. a) –6 × –7 = 42 b) 9 × –7 = –63

1.4 Multiplication

1. a) 34 b) 0 c) 129 d) –8
2. a) 7 b) 6 c) 12 d) 12
3. a) –25 and 10 b) –25 and –5 c) –60 and –5
4. a) 646 b) 350 c) 1854 d) 2728
5. £425
6. 64 × 12 = 768 p = £7.68
7. 18 × 12 = 216 m²
8. a)

×	40	7
20	800	140
5	200	35
total	1000	175

   b) 1175
9. a) She can subtract 50 from 1000 b) 50 × 19 = 950
10. He has not put a zero in the units column for the second row (so he has multiplied 62 by 3 rather than by 30).
    The correct answer is 62 × 37 = 2294

1.5 Division

1. a) 45 lots of 19 is 855, so 855 divided by 19 equals 45
   b) The product of 36 and 42 is 1512, so 1512 divided by 42 equals 36
   c) 58 multiplied by 24 equals 1392, so 1392 divided by 24 equals 58
2. a) 35 b) 43 c) 37 d) 21
3. a) 25 b) 38 c) 30 d) 52
4. a) 21 r 5 b) 14 r 6 c) 21 r 17 d) 28 r 3
5. The remainder must be less than the divisor
6. 33 packets
7. 18 people
8. a) 21 pieces b) 45 cm
9. 12 coaches
10. 1258
11. 544 divided by 16 = 34 remainder 0
12. 221 (17 × 13)

1.6 Priority of operations

1. a) 15 b) 8 c) 7 d) 8
2. a) 42 b) 3 c) –28 d) 98
3. a) 5 b) 29 c) 5 d) 5
   part b) is the odd one out.
4. Tamin is wrong; 6 + 5 × 8 = 6 + 40 = 46.
   He has done the addition first and then the multiplication.
5. a) (20 – 10) ÷ 2 = 5 b) 2 × (7 + 2) = 18
   c) 24 ÷ (8 – 2^2) = 6 d) 10 × (12 – 8) + 2 = 42
6. a) 4 + 6 × 3 < (4 + 6) × 3 b) 5 × 6 ÷ 2 = 5 × (6 ÷ 2)
   c) 10 – 3 × 2 < (10 – 3) × 2 d) 9 + 1^2 < (9 + 1)²
7. c) 20 – (3 × 2 + 5)
8. The calculation only multiplies 13 by 2; she needed to add 13 and 18 first before multiplying by 2.
   The correct calculation is (18 + 13) × 2 (= 62)
9. a) 45 b) 4 c) 4 d) 5
10. a) (3 + 0) × (10 – 7) = 9 b) 3 × 1 ÷ (10 – 7) = 1
    c) 3 ÷ (0 + 10 – 7) = 1 d) 3 – (0 × 10 – 7) = 10

Section 2

2.1 Square numbers

1. a) 25 b) 49 c) 121 d) 169
2. a) 6 b) 9 c) 12 d) 10
3. a) 16 b) 2 c) 13 and –13 d) 64
4. 8
5. a) $14^2 > 144$ b) $\sqrt{121} < 15^2$
   c) $3^2 = \sqrt{81}$ d) –6 < the negative square root of 25
6. $(-2)^2 = 4$ $\sqrt{100} = -10$ $(-3)^2 = 9$ $0^2 = 0$
7. Answers to a) and c) are not integers
8. a) 10 b) 16 c) 9, 81
9. a) the square root of 16 is 4 cm not 8 cm
   b) the length of the square = 4 cm and the perimeter is 4 × 4 = 16 cm
10. a) sometimes true
    e.g. $1^2 = 1$ (square is not greater) but $2^2 = 4$ (square is greater)
    b) sometimes true e.g. $2^2 = 4$ (is even) but $3^2 = 9$ (is odd)
    c) always true e.g. $\sqrt{0} = 0$, $\sqrt{25} = 5$ or –5 (so a square root can be positive, negative or zero)
    d) never true e.g. anything squared is always positive

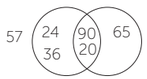
2. 2 Index notation

1. a) 3^3 b) 4^5 c) 2^4 d) 5×10^2
2. a) 1 b) 125 c) 1000 d) 64
3. a) 2 b) 3 c) 1 d) 4
4. a) 200 is the odd one out.
   All numbers are square numbers apart from 200
   b) 9 is the odd one out. All numbers are cube numbers apart from 9
5. a) 5^2 (25) is larger than 4 b) 10^3 (1000) is larger than 27
   c) 4^5 (1024) is larger than 625 d) 10^1 (10) is larger than 3
6. $2^5 = 32$ $\sqrt[3]{1000} = 10$ $\sqrt{196} = 14$ $\sqrt{121} = 11$ $\sqrt[3]{1} = 1$
7. a) 169 b) 64 c) 3 d) 3
8. a) False $1^{10} < 10^1$ b) false $2 \times 2 \times 2 = 2^3$
   c) false $\sqrt[3]{1} = 1$ only d) true
9. a) e.g. $\sqrt[3]{1} = \sqrt{1}$ b) e.g. $64 = 8^2 = 4^3$
10. Square number = 196 (= 16^2) and
    Cube number = 27 (= 3^3). 196 – 27 = 169 (= 13^2)

2.3 Laws of indices

1. a) 4^5 b) 10^6 c) 5^6 d) 11^8
2. a) 3^3 b) 2^5 d) 11^5 d) 7^0
3. a) 2^6 b) 2^{12} c) 3^{15} d) 3^{30}
4. part a) $(10^2)^4 = 10^8$ and part c) $5^3 \times 2^3 = 1000$; do not equal 10^6
5. a) $2^5 \times 5^2$ b) $3^5 \times 5^4$ c) $5^4 \times 2^7$ d) 10^5
6. Part d) is the odd one out – as it is the only one which is correct.
7. a) No, she is not correct. The answer is 2^5
   b) No, he is not correct. The answer is 8^8
8. a) * = 5 b) * = 5 c) * = 4
9. a) Mistakes: $(2 + 1)^3 = 3^3 = 27$ d) Mistakes: $5^6 \div 5^6 = 5^0 = 1$
10. a) $10^0 = 1^5$ b) $9^3 = 3^6$ c) $2^4 = 4^2$

2.4 Prime numbers

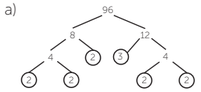
1. 37 and 79 are prime
2. 

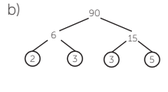
3. a) 49 (odd number) b) 51 (not divisible by 5)
   c) 28 (not divisible by 6) d) 3 (not divisible by 9)
4.

	Divisible by 2	Divisible by 3
Prime Numbers	2	3
Not prime numbers	10, 32	9, 45, 63

5. a) False b) true c) true d) true
6. a) False. 2 is prime number and is even.
   b) False. 2 is the smallest prime
   c) False. 97 is the largest prime under 100. 99 is not prime.
   d) True
7. a) integer answer b) not integer answer; not divisible by 9
   c) integer answer d) not integer answer; not divisible by 3
8. 90
9. a) sometimes true b) sometimes true
   c) never true d) never true
10. 2, 5, 11

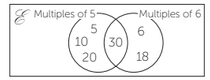
2.5 Factors

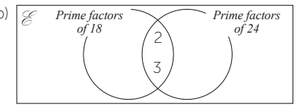
1. a) 2^6 b) $2^3 \times 3^2$ c) 2×3^4 d) $2^2 \times 3 \times 5^2$
2. a) 28 b) 36 c) 40 d) 180
3. a) 

   b) 

4. 4 and 1 are not prime factors. 24 = 2 × 2 × 2 × 3
5. a) 40 = 2 × 2 × 2 × 5 b) 32 = 2 × 2 × 2 × 2 × 2
   c) 52 = 2 × 2 × 13 d) 105 = 3 × 5 × 7
6. Isabella is correct. Jake has not used index form and Ben has not multiplied.
7. 48 = 2 × 2 × 2 × 2 × 3 = $2^4 \times 3$
8. a) 33 = 3 × 11 and b) 1 is not a prime factor
9. a) true since 2 × 3 =6
   b) true since $2^2 = 4$
   c) false; it is a multiple not a factor
   d) true since $2^3 \times 5 = 40$ and so is a factor of itself.
10. a) e.g. $32 = 2^5$ b) e.g. $500 = 2^2 \times 5^3$
    c) e.g. 48 = 3 x 2^4 and 700 = $2^2 \times 5^2 \times 7$

2.6 Multiples and LCM

1. a) 4, 8, 12, 16, 20 b) 10, 20, 30, 40, 50
   c) 15, 30, 45, 60, 75 d) 30, 60, 90, 120, 150
2. 

3. 24
4. a) 18 = 2 × 3 × 3 24 = 2 × 2 × 2 × 3
   b) 

   c) LCM (18, 24) = 72
5. a) LCM (6, 8) = 24 b) LCM (9, 12) = 36
   c) LCM (10, 25) = 50 d) LCM (13, 7) = 91
6. LCM (2, 3, 5) = 30
7. e.g. 9 or 18 or 45
8. e.g. 7 or 14 or 42
9. 13 times including midnight and noon.
10. 5, 7, 13
11. Each 30 seconds, Sandi has done 5 and Joel has done 4. So 5 × 60 ÷ 30 = 10 times.
12. Mr Green bakes 15 batches of brownies.
    Miss Silver bakes 7 batches of cookies. They will make 210 of each.

7. $y^2 - 2y - 8 = 7$
$y^2 - 2y - 15 = 0$
$(y - 5)(y + 3) = 0$
$y = 5$ and -3

8. Intersects x-axis at (6,0) and (–5,0), y-axis at (0,–30)

25.5 Working with formulae

1. a) £11 b) £17.50 c) £28.30
d) 40 kilometres e) 102 kilometres

2. 56 cm^2

3. a) 10 cakes b) 19 cakes c) 32 people
d) 20 people require 10 cakes and 40 people require 15 cakes (the +5 is the issue)

4. Jasmine needs to divide the whole of $t + 3$ by 4.
Answer should be $n = \frac{t+3}{4}$

5. a) $t = \frac{d}{7}$ b) $n = m - 9$ c) $h = \frac{c-10}{6}$
d) $g = 10 - h$ e) $x = 2(y + 5)$ or $x = 2y + 10$

6. $h = \frac{c - xy}{12}$

7. a) $x = \frac{y+2}{4}$ b) $x = \frac{2y}{9}$ c) $x = \sqrt{y}$
d) $x = \sqrt{\frac{7-y}{4}}$ e) $x = \sqrt{\frac{3y+1}{3}}$

8. $b = \frac{2A}{h} - a$

9. $r = \sqrt{\frac{2A}{\pi}}$

10. $x = \sqrt{4\pi - 7y}$

25.6 Limits of accuracy

1. a) 18 453 b) 18 450 c) 18 500
d) 18 000 e) 20 000

2. Gerry has not found a number that is close to what there was initially, but has rounded the 4 up to a 5 correctly
Paul has rounded to 2 decimal places not 2 significant figures
Natalie should have rounded the 4 up to 5.
Correct answer is 1500

3. 135

4. Any number from 399.5 up to but not including 400.5.
e.g. 399.5, 400.4

5. 43 minutes 29 seconds

6. a) 75 s ≤ actual value < 85 s
b) 12 250 m ≤ actual value < 12 350 m
c) 33.5 cl ≤ actual value < 34.5 cl
d) 12.25 cm ≤ actual value < 12.35 cm

7. Greatest bicycle length: 154.9 centimetres
Least shed length: 1.5 m = 150 cm
The bicycle is not guaranteed to fit in the shed

8. a) 20.689 655 172 b) 20.7 c) 20
d) Truncated to 20 is the most sensible degree of accuracy because you can't get part of a coin.

9. 90 ÷ 7 = £12.85714286
Clare's method would give everyone £13 (but £13 × 7 = £91 so someone will get £1 less than everyone else)
Suggest truncating to 2 decimal places (nearest penny) as everyone will receive the same, with just 5p left over. Or, truncate to 1 decimal place, give everyone £12.80 and keep the extra 40p..

10. 74.75 cm^2 ≤ Area < 93.75 cm^2

CALCULATOR HACKS

1 Prime factorisation

SHIFT – FACT

Use this to find the prime factors of a number.

Example
To find the prime factors of 360:

Key sequence

360 = SHIFT °'''

Calculator screen shows

360
$2^3 \times 3^2 \times 5$

2 Using table mode

Mode 3

For most calculations, the calculator will be in Mode 1 (COMP)

Mode 3 – TABLE can be used to fill in tables, for graphs.

Remember! Ensure that you go back to MODE 1 after filling in the table

Example

Complete the table of values for

$$y = x^2 + 2x + 4$$

x	-4	-3.5	-2	-1	0	0.5	2
y							

Key sequence

MODE 3	ALPHA)	x^2 +2 ALPHA) +4	= Start?	–4 =	End? 2 =	Step? 0.5 =
_____	_____	_____	_____	_____	_____	_____

The prompts Start? and End? require the inputs of the smallest and the largest value in the table (–4 and 2 in the example)

Example

The calculator gives you all y values for steps of 0.5 in x values.

The table on the exam paper shows unequal step sizes, so you must ensure that you read the correct y values from the calculator, to match the x values on the exam paper.

x	-4	-3.5	-2	-1	0	0.5	2
y	12	9.25	4	3	4	5.25	12

Calculator screen shows

	X	F(X)
1	-4	12
2	-3.5	9.25
3	-3	7

-4.

3 Entering mixed numbers

Example

When calculating

$$1\frac{1}{2} - \frac{3}{4}$$

a common error is to enter 1 then press and enter ...

$$\frac{1}{2}$$

Instead, key the following sequence...

SHIFT

You should see three boxes on the screen

Key sequence

1 ▶ 1 ▶ 2 ▶ - 3 ▶ 4 =

The arrow in the key sequences shown is typically a large white button marked **REPLAY**

Calculator screen shows

$1\frac{1}{2} - \frac{3}{4}$ $\quad \frac{3}{4}$

4 Cancelling fractions

Express the following faction in its simplest form:

$$\frac{18}{27}$$

(Note: This may be the last part of a question on the calculator paper.)

Key sequence

18 ▶ 27 = $\frac{2}{3}$

5 Are you in the correct TRIG mode for the exam?

Marks can be lost for not being in the correct trig. mode.

Quick check
When you key in the sequence

SIN (30)

Do you see this?

sin(30)

$\frac{1}{2}$

YES - you are in the correct MODE
NO - you need to change the MODE

Key sequence

To change the mode, key the following

SHIFT SETUP Option 3 3Deg

Go back and follow the instructions for checking your calculator is in the correct mode.

CORE SKILLS AND COMMON MISTAKES

There are six core skill areas in your maths papers. Each of these areas are covered within this book and can be identified for each topic by the different tab colours of the specification references.

This is a summary of common errors made in past examinations. Avoid the same pitfalls and learn from the mistakes of others!

Number N

1. Students' responses to some questions have many arithmetical errors, mainly in calculations requiring division.
2. Even on the calculator papers, some students use incorrect non-calculator methods, indicating they had no calculator (or were unable to use one).
3. Rounding to a given number of significant figures poses problems for some students. Be confident with this.
4. Many students still struggle with the concept of dividing fractions.
5. Money problems are tackled well, but questions involving other units, or a change of unit are poorly attempted.
6. Be prepared in your understanding of union or intersection, and their association with a Venn diagram.
7. Many misunderstandings relating to time were noticed, particularly when using a timetable or journey planning.
8. Students commonly do not recognise that an instruction to estimate an answer should trigger them to apply rounding. Any attempt to apply a complex calculation results in zero marks being awarded. Note that any attempt to round will gain some marks, not necessarily just to 1 significant figure.

Algebra A

1. When negative values are involved, students' performance is generally weaker. For example, drawing a graph of $y = 1 - 4x$, or in calculating the values for a quadratic where the x values are negative. Students should be more practised in using the symmetrical properties of a parabola to check their curve.
2. Rearranging formulae remains a weakness of many Foundation students and should be practised much more.
3. There seems to be little understanding of the relationships between equations and their graphs, for example, using the values of m and c on parallel graphs, when finding an equation of a straight line.
4. Methods of solving equations vary, but those who use the 'equation balancing' method tend to achieve more marks. Students should be reminded that it is rare to achieve full marks using trial and improvement methods and this method should be avoided. Students should also take care in using the correct order of operations.

Ratio, Proportion and Rates of Change R

1. Many candidates attempt non-calculator methods for finding percentages on the calculator paper, leading to incorrect answers. Use the calculator for these questions or to check your answers.
2. On the non-calculator paper, percentages are mostly attempted by building up to the required percentage, and often candidates have difficulty in piecing together the parts.
3. Students have great difficulty when attempting questions using linked ratios (e.g. Given a : b and b : c, find a : c)
4. There are some instances of students failing to simplify ratios, even when asked to do so.
5. Scale diagrams are a weakness for many candidates.
6. Compound measures, such as speed, density, pressure and any context involving proportional units are frequently misunderstood.

Geometry and Measures G

1. Recall of essential formulae remains a weakness, particularly those for areas of a triangle and trapezium and those related to a circle.
2. The use of correct mathematical language, for example, in geometrical reasoning and in transformation geometry is commonly seen. Students must be reminded that non-technical language will not gain any marks.
3. The handling and conversion of units is commonly misunderstood by Foundation candidates. Students, at both Foundation and Higher level, need to be reminded that there is usually one question in which they must state their units.
4. Mensuration work on problem solving continues to be challenging for students. They often mis-read the question and therefore miss out essential parts of the process for gaining a complete solution.
5. Finding the sum of the interior angles of a polygon requires more work, as many students assume it is 360° regardless of the number of sides of the polygon. It is calculated as the number of sides – 2 x 180°.

Probability P Statistics S

1. When interpretation of a composite bar chart has been tested, students demonstrated significant weakness in interpreting it.
2. There is still evidence that protractors are being used inaccurately, or that students do not have a protractor, when drawing / interpreting pie charts.
3. While there is an improvement in writing criticism on statistical diagrams, students often write conflicting remarks, or comments that were too vague.

NOTES, DOODLES AND EXAM DATES

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Doodles

Exam dates

Paper 1:

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Paper 2:

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Paper 3:

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EXAMINATION TIPS

When you practise examination questions, work out your approximate grade using the following table. This table has been produced using a rounded average of past examination series for this GCSE. Be aware that boundaries vary by a few percentage points either side of those shown.

GCSE Maths: Foundation

Grade	5	4	3	2	1	U
Paper 1F (%)	73	59	44	29	14	0
Paper 1F (%)	71	58	43	28	13	0
Paper 1F (%)	68	54	40	26	13	0

Overall grade	5	4	3	2	1	U
F Tier (%)	71	58	43	28	13	0

1. Read questions carefully. This includes any information such as tables, diagrams and graphs.
2. Remember to neatly and clearly cross out any work that you do not want to be marked. Do not scribble over it, rub it out or render it illegible in some other way.
3. Learn how to use your calculator, compasses and protractors correctly and take them to the exams. There is no calculator allowed in Paper 1F.
4. Show your workings. There are marks awarded for workings out on some questions, even if the answer is corrrect. Even the most basic of calculations or steps must be shown. This is particularly true of the calculator papers.

   These questions will typically state in the question:

   - "You must show all your working."
   - "Give reasons for your answer."
   - "Prove..."
5. Avoid using multiple methods to answer the same question. Examiners are instructed to award 0 marks for workings that are ambiguous, or where it is not clear which method leads to the answer given.

   If you change your mind on a method, you should cross out the previous working and show the intended method clearly.
6. Presentation matters. Good written communication helps the examiner to award you marks.

   Common issues include include:

   - Illegible handwriting can mean examiners don't award you marks if they can't confidently read your answers.
   - Some students write answers in a foreign language and therefore work cannot be marked - all answers are expected to be in English.
   - The numbers 4 and 9 are more commonly written ambiguously, also 1 and 7.
   - Over-writing to correct mistakes is becoming more common. Students are reminded again to cross out and re-write their answers.
7. Check through your answers if you have spare time. It is very easy to make a silly mistake that you could easily correct for a few extra marks. It might make the difference between two grades. If you go wrong somewhere, you may still be awarded some marks if the working out is there. It is also much easier to check your answers if you can see your working out. Remember to give units when asked to do so.

Good luck!

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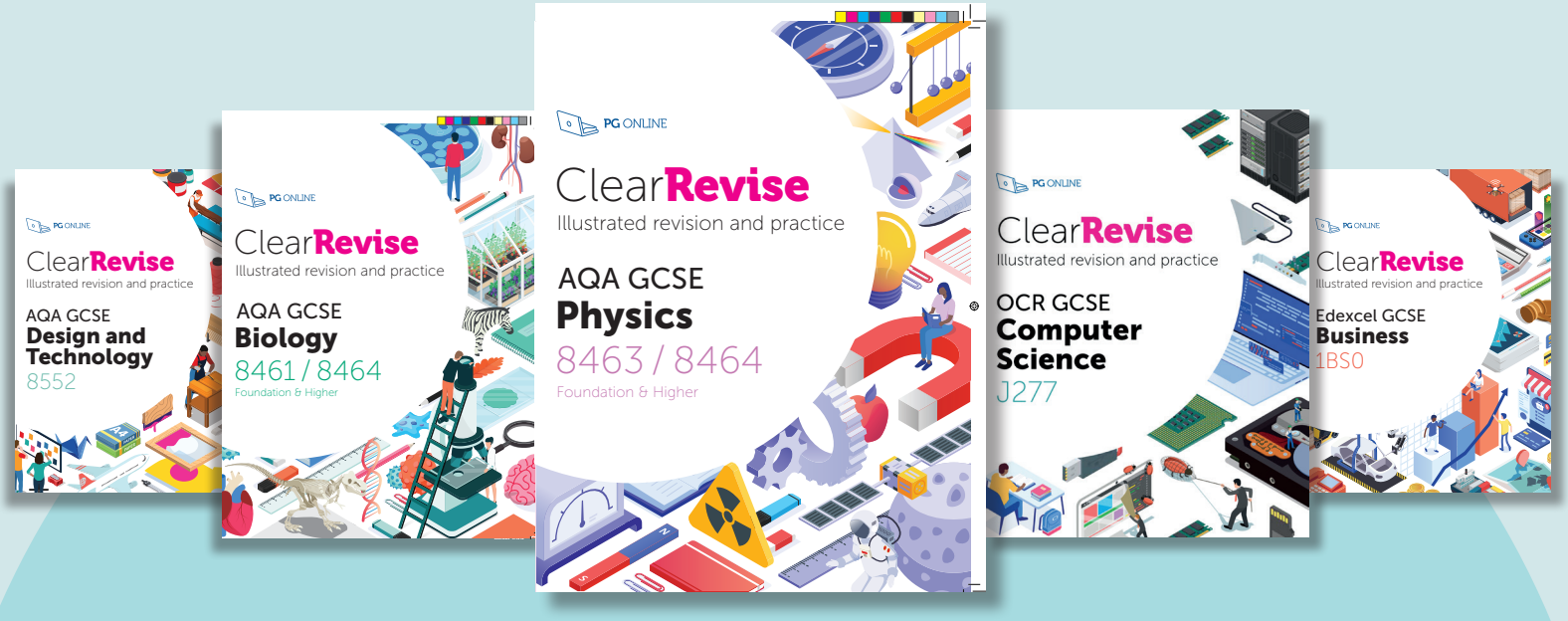

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